Master internship on
Student t-Processes under Inequality Constraints

Inequality constraints. Accounting for physical constraints in regression is crucial to ensure more realistic predictions. In biology, predictions must remain non-negative when modeling protein concentrations. In aerodynamics, the lift force of an airfoil is expected to increase monotonically with airspeed within a certain range. In both cases, models should account for the constraints to prevent implausible outcomes, and to improve interpretability and robustness.

Gaussian processes (GPs) and constraints. GPs have gained popularity as nonparametric Bayesian priors for modeling functions, offering both theoretical and computational advantages [1]. They have been successfully applied in various fields, including machine learning, engineering and geostatistics. Constrained approaches have been explored in regression tasks [2, 3], point process modeling [4], and parameter inference [3, 5]. We refer to [6] for a comprehensive survey on constrained GPs.

To the best of our knowledge, the finite-dimensional approach studied in [2, 3, 7] is the only method capable of handling various types of inequality constraints while ensuring they are satisfied everywhere. Theoretical properties have been investigated, demonstrating convergence of the MAP estimate [8], asymptotic consistency and normality of the likelihood estimator [3, 5], and error bounds for predictions [9]. Despite its success, the finite-dimensional approach has certain limitations. Like unconstrained GPs, constrained GPs can be overly sensitive to outliers.

Upgrading to Student-t processes (TPs). TPs can be seen as a generalization of GPs [10]. From a finite-dimensional point of view, as the shape parameter ν of the Student-t distribution gets larger, the tails of the distribution converge to the Gaussian tails. When ν is small, the distribution becomes more heavy-tailed. This added flexibility offers two main benefits. First, extreme values can be made more likely, providing better risk estimates in many settings. Second, the posterior variance increases or decreases depending on the variance of observed data sample values, and not only on their location, as is the case in GP. Numerical experiments in [10, 11] have tested the performance of (unconstrained) TPs, demonstrating their superior performance over (unconstrained) GPs in both regression and global optimization.

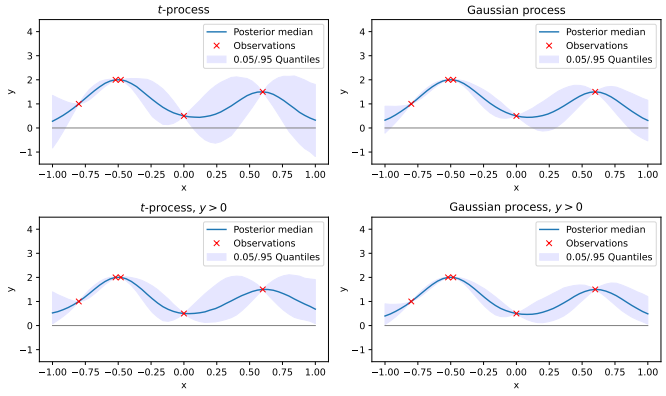


Figure 1: Predictions obtained from TPs under positivity constraints. Given five observations of a function $y = f(x)$ known to be positive, we sketch the posterior predictive distributions of TPs and GPs, with and without positivity constraints. Note that the probability wrongly assigned to negative regions (upper row) is higher for TPs due to its fatter tails, and thus the relevance of imposing positivity.

Objectives of the master internship. The internship aims to develop novel TP frameworks under inequality constraints, to advance the state-of-the-art in computational and/or theoretical aspects. For numerical implementations, one will consider evaluating outliers-robust models in scenarios including regression and Bayesian optimization, both of great interest in machine learning and engineering applications. The development of open-source Python or R codes is expected.

Organizational details and supervising team. The master internship is funded for 4-6 months and is expected to begin in late winter or early spring 2027. It will take place at the institute of mathematics IMAG, Université de Montpellier (UM), under the supervision of:

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As the internship is funded by the PHC Maïmonide project 53907UA, collaborations with other members of the project will be encouraged.

How to apply? Applications are considered until the position is filled. Please send a CV (in English or French), an application letter and the grade transcripts (bachelor and master level) to the supervising team. The candidate should have master-level skills in mathematics, statistics or machine learning. Outstanding Python or R programming skills will be appreciated.

References

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